EE 102—Digital Logic and Design
Lecture #5

Combinational Circuits

Combinational Circuits

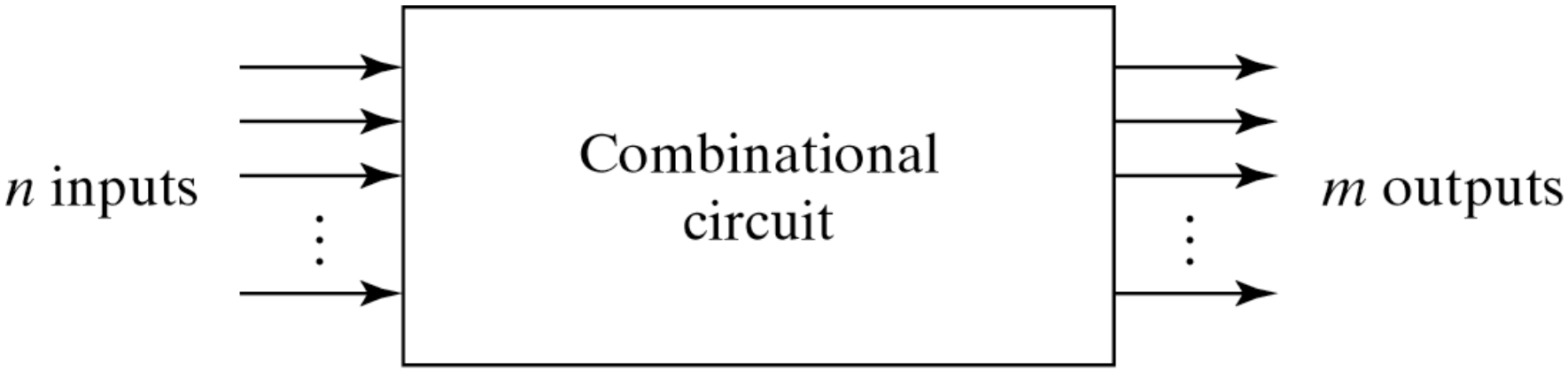


Fig. 4-1 Block Diagram of Combinational Circuit

4-2. Analysis procedure

- To obtain the output Boolean functions from a logic diagram, proceed as follows:

1. Label all gate outputs that are a function of input variables with arbitrary symbols. Determine the Boolean functions for each gate output.

2. Label the gates that are a function of input variables and previously labeled gates with other arbitrary symbols. Find the Boolean functions for these gates.

4-2. Analysis procedure

3. Repeat the process outlined in step 2 until the outputs of the circuit are obtained.

4. By repeated substitution of previously defined functions, obtain the output Boolean functions in terms of input variables.

Example

$F_2 = AB + AC + BC$; $T_1 = A + B + C$; $T_2 = ABC$; $T_3 = F_2'T_1$;

$F_1 = T_3 + T_2$

$F_1 = T_3 + T_2 = F_2'T_1 + ABC = A'BC' + A'B'C + AB'C' + ABC$

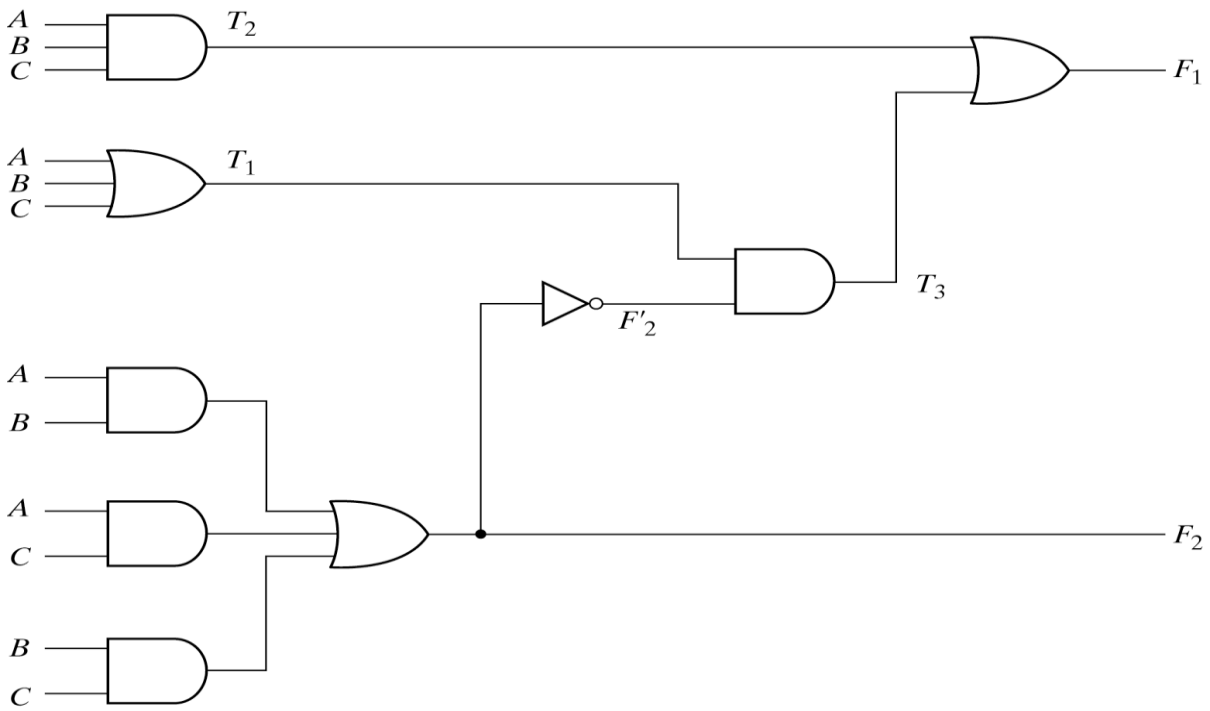


Fig. 4-2 Logic Diagram for Analysis Example

Designing Combinational Circuits

In general we have to do following steps:

1. Problem description
2. Input/output of the circuit
3. Define truth table
4. Simplification for each output
5. Draw the circuit

4-4. Binary Adder-Subtractor

- A combinational circuit that performs the addition of two bits is called a half adder.
- The truth table for the half adder is listed below:

Table 4-3
Half Adder

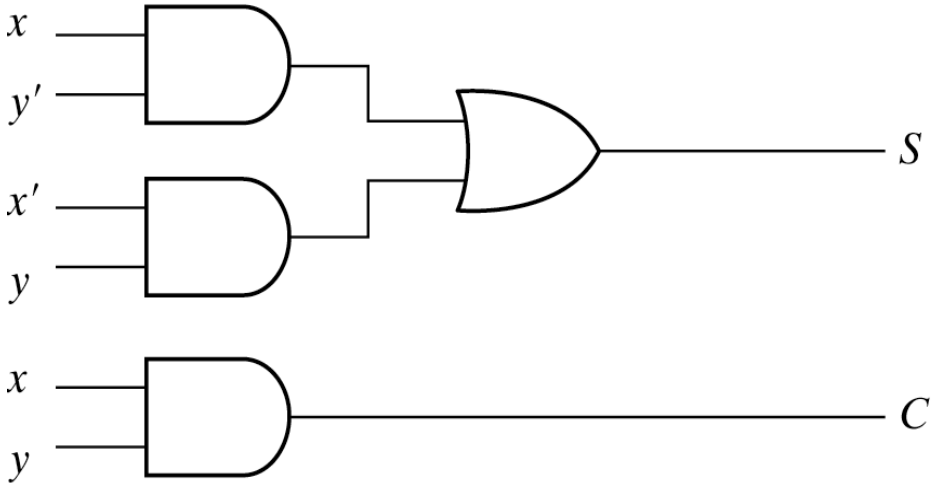
x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

S: Sum
C: Carry

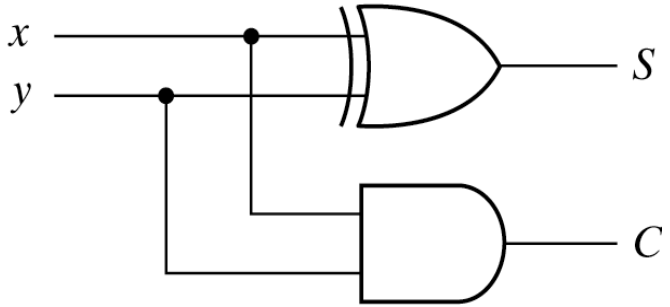
$S = x'y + xy'$

$C = xy$

Implementation of Half-Adder



(a) $S = xy' + x'y$
$C = xy$



(b) $S = x \oplus y$
$C = xy$

Fig. 4-5 Implementation of Half-Adder

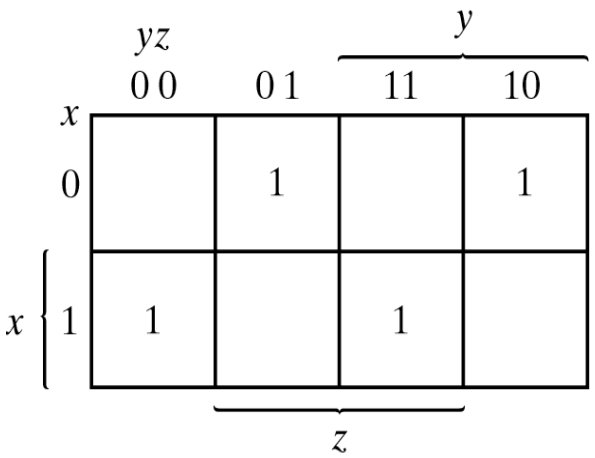
Full-Adder

- One that performs the addition of three bits(two significant bits and a previous carry) is a full adder.

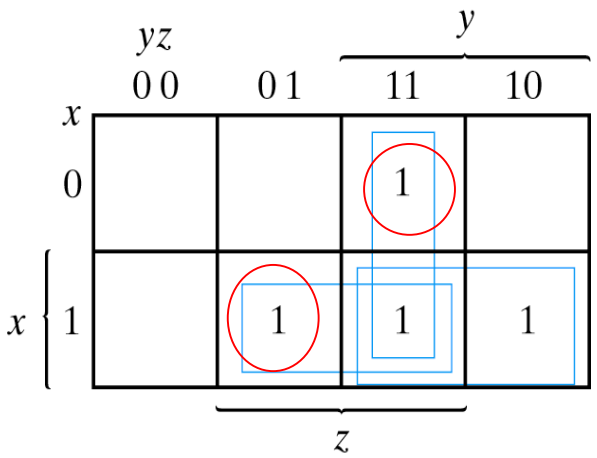
Table 4-4
Full Adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Simplified Expressions



$S = x'y'z + x'yz' + xy'z' + xyz$



$C = xy + xz + yz$
$= xy + xy'z + x'yz$

Fig. 4-6 Maps for Full Adder

S = x'y'z + x'yz' + xy'z' + xyz

C = xy + xz + yz

Full adder implemented in SOP

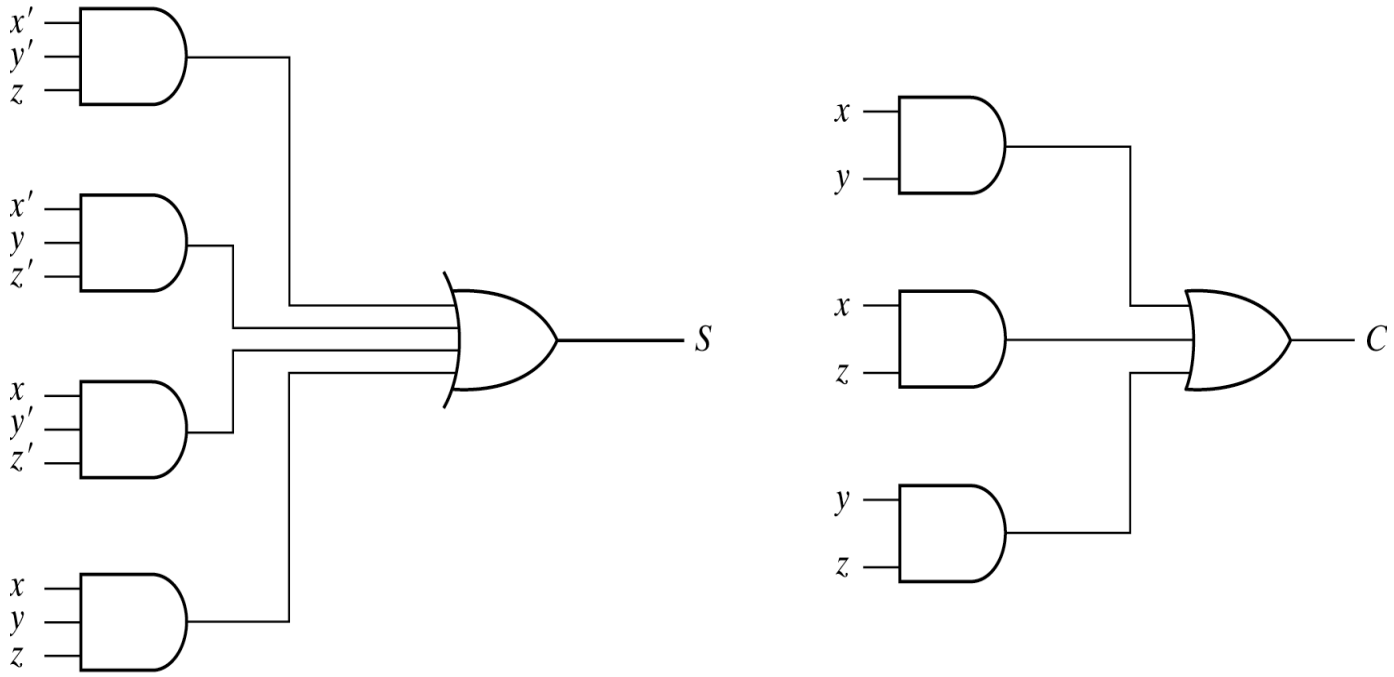


Fig. 4-7 Implementation of Full Adder in Sum of Products

Another implementation

- Full-adder can also implemented with two half adders and one OR gate (Carry Look-Ahead adder).

$$S = z \oplus (x \oplus y)$$
$$= z'(xy' + x'y) + z(xy' + x'y)'$$
$$= xy'z' + x'yz' + xyz + x'y'z$$
$$C = z(xy' + x'y) + xy = xy'z + x'yz + xy$$

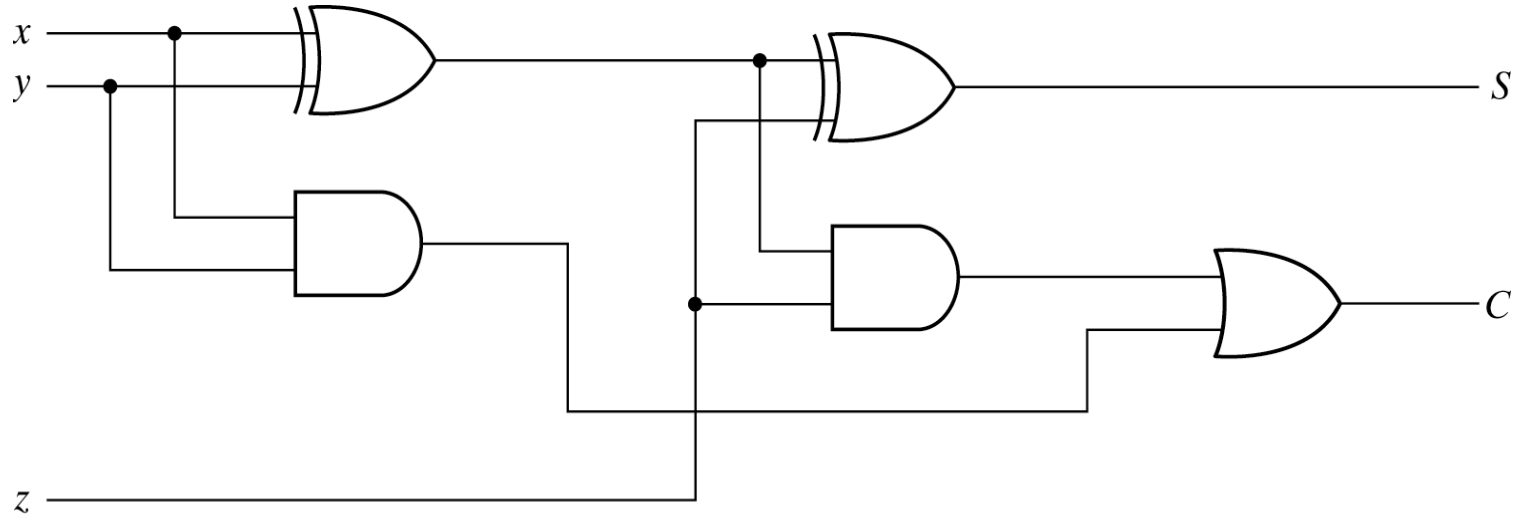


Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate

Binary adder

- Binary adder that produces the arithmetic sum of binary numbers can be constructed with full adders connected in cascade, with the output carry from each full adder connected to the input carry of the next full adder in the chain
- Note that the input carry C_0 in the least significant position must be 0.

Binary adder

- This is also called Ripple Carry Adder ,because of the construction with full adders are connected in cascade.

Subscript i:	3	2	1	0	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

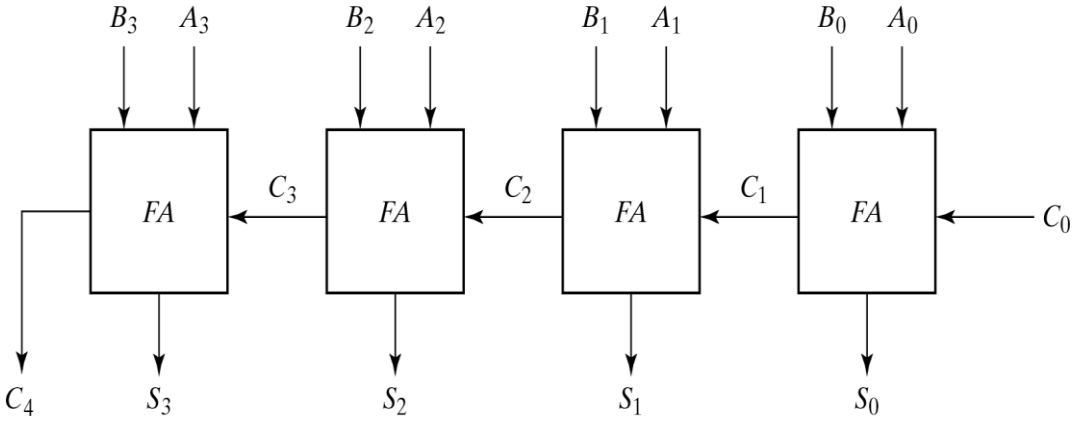


Fig. 4-9 4-Bit Adder

Binary Adder

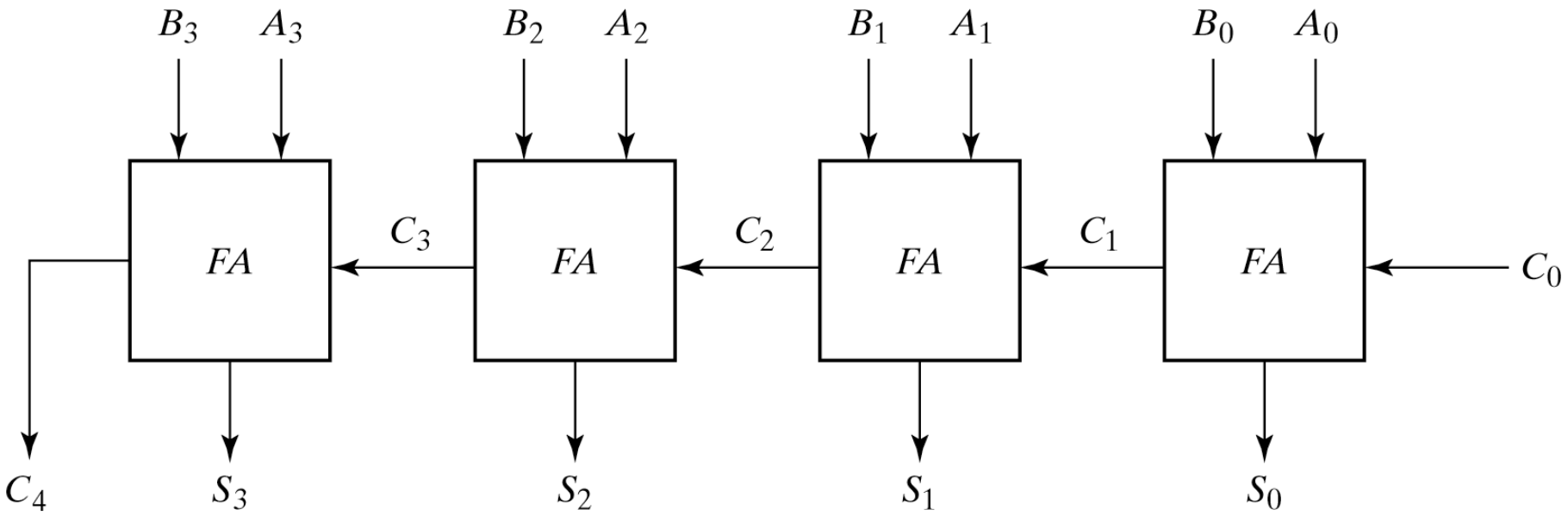


Fig. 4-9 4-Bit Adder

Binary subtractor

- The subtraction A - B can be done by taking the 2's complement of B and adding it to A. The 2's complement can be obtained by taking the 1's complement and adding 1 to the least significant pair of bits. The 1's complement can be implemented with inverters, and a 1 can be added to the sum through the input carry.

Binary subtractor

M = 1→subtractor ; M = 0→adder